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ON SUPERNOVA HYDRODYNAMICS

Recently, Colgate (1968) has criticized an investigation of gravitational collapse of non-rotating massive stars by the author (Arnett 1967) in which energy transport by electron-type neutrinos and antineutrinos was found to be much less efficient in massive stars ($M_{\text{core}} \gtrsim 8\ M_\odot$) than estimated by Colgate and White (1966; hereinafter referred to as "CW"). Mathematical models of these stars would not explode by this mechanism. The purpose of this Note is to reply to Colgate's criticisms and point out some weaknesses in the analysis given in CW.

I. CRITICISM OF ANALYSIS OF COLGATE AND WHITE

In order to gain a proper perspective, we begin by discussing three aspects of the CW calculation which the author found to be unsatisfactory; they are: (*a*) approximate form of the equation of state which underestimates pressure due to nucleons, (*b*) restrictive treatment of energy transfer by neutrinos and antineutrinos, and (*c*) neglect of the effects of muon-type neutrinos and antineutrinos.

a) *Equation of State*

In their equation (51), page 651, Colgate and White (1966) use a pressure term which is appropriate for a non-relativistic gas of ionized ^{56}Fe and electrons but is too small by a factor of 2 for a pure neutron gas. The error is larger if contaminants (such as electrons and positrons) are admitted. Calculations by the author (nearly identical with those of Arnett [1967] but which purposely included this error) reveal a collapse in which each zone falls individually from some critical density ($\rho \lesssim 10^{11}$ g cm^{-3}) onto a dense core ($\rho \gtrsim 4 \times 10^{14}$ g cm^{-3}). This sort of behavior was found in CW. Finite difference techniques are no longer valid for any reasonable number of mass zones. The situation may be visualized as rather like pushing a line of books off the end of a table; each falls separately.

Even in the case of zero temperature (all particles are degenerate) it appears that the pressure for densities in the range

$$10^{11} < \rho < 10^{14} \text{ g cm}^{-3}$$

has been underestimated by CW. The Salpeter (1960) equation of state for a degenerate nucleon gas, which was used by CW, appears to be dominated to an unusual degree by attractive parts of the nucleon-nucleon potential. In particular, it seems to be quite different from the equations of state predicted by the Skyrme and the Levinger-Simmons potentials (Tsuruta and Cameron 1966). An examination of the problem by Hansen (1967) indicates the possibility of an error in the position of a decimal, the correction of which results in better agreement with other equations of state. A crude estimate by the author using measured phase shifts is in agreement with this conclusion.

b) *Restrictive Calculation of Energy Transfer*

In the paper by Arnett (1967) the *coupled* equations of hydrodynamics and equilibrium radiative diffusion of energy were solved numerically. Because the region transparent to neutrinos ($\tau \lesssim \frac{2}{3}$) was treated separately, it was possible to include the energy transfer assumptions of CW as a special case. Thus if the physical conditions were as they assumed, their results would have been reproduced. This was the case for less massive stars ($M_{\text{core}} \sim 2\, M_\odot$), for which the results of CW and Arnett are in qualitative agreement. For more massive stars, their assumption that the neutrino-emission surface coincides with the shock front formed by matter falling on a quasi-static central region of nuclear densities is invalid.

In order to see why this is so, let us examine in some detail the assumptions involved in the CW treatment of neutrino energy transfer. In order to estimate the neutrino flux (in what follows, the term "neutrinos" refers to electron-type neutrinos and antineutrinos unless explicitly qualified) from the shock set up by matter falling on a dense, quasi-static core, we must know the velocity with which matter enters the shock front. When the pressure forces are *negligible* in the infalling matter, we may relate the kinetic energy of free fall to the gravitational potential at the surface of the dense core. This is determined in turn by the mass and radius of the core, which may be found from a knowledge of *the equation of state* and the structure of the pre-supernova star at the onset of instability. Since the equation of state will depend, in general, upon the temperature, the previous flux history is required. If there is considerable energy transfer from behind the shock into the infalling matter, thermal effects may produce a pressure gradient which modifies the flow. Although this preheating of the infalling matter is a necessary consequence of the analysis of CW, it was not considered properly in their numerical models.

Colgate and White have used a very approximate form for energy loss by the nuclear URCA process (their equation [53]) which cools the matter faster than gravitational compression can heat it for densities $\rho \gtrsim 10^{11}$ g cm^{-3}. Even with a corrected equation of state, this energy loss rate is so violent as to cause each zone to fall individually to high density, in the unrealistic manner mentioned above. CW have correctly argued that quite high temperatures will develop behind the core shock, although their numerical results for these temperatures were in error because of their coarse zoning in mass and erroneous equation of state. To avoid this computational impasse, they regarded the radius of the core shock and the radius of the neutrino emission surface ($\tau = \frac{2}{3}$) to be *identical. No justification for this assumption has been given.* Half of the energy lost by the nucleon URCA process inside this radius was considered to be reabsorbed by matter outside this radius. For non-pathological initial conditions, this *insures* that an explosion will result.

In particular, it is incorrect to assume in general that the flux of kinetic energy entering the core shock is equal to the flux of energy in electron-type neutrinos emitted from the core shock. This gives an *upper* limit for the electron-type neutrino flux. Arnett (1967) has shown that for massive cores this energy may be stored for a time greater than the collapse time scale ($t > 5 \times 10^{-3}$ sec) or lost by muon-type neutrino emission.

c) Neglect of Muon-Type Neutrinos

The effects of weak interactions involving muons were neglected in CW. At low temperatures and densities, the number density of muons is low, so that most interactions are with electrons or nucleons rather than with other muons. For the temperatures encountered by Arnett (1967), $kT \ll m_\mu c^2$, so that the weak interactions

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \, ,$$

$$\mu^+ \rightarrow e^+ + \nu_e + \nu_\mu \, ,$$

$$\mu^+ + e^- \rightarrow \bar{\nu}_\mu + \nu_e \, ,$$

all produce muon-type neutrinos of average energy $\epsilon < m_\mu c^2$. Because of the extremely low number density of muons in the matter outside the core shock, effective energy transfer by muon-type neutrinos requires an energy $\epsilon > m_\mu c^2$ to overcome the threshold of $Q \approx m_\mu c^2$ for the most likely inverse reactions. The reaction

$$\mu^- + \mu^+ \rightarrow \nu_\mu + \bar{\nu}_\mu$$

can give neutrinos of energy $\epsilon > m_\mu c^2$, but it appears that, for the temperatures encountered in even the very massive stars ($M_{\text{core}} = 32\, M_\odot$), this process is much less important than those discussed above. This preliminary analysis indicates that muon-type neutrinos escape without interaction and that the suggestion by Colgate (1968) that energy transfer by muon-type neutrinos might occur does not apply here.

II. CRITICISM BY COLGATE

Colgate (1968) has criticized the paper by Arnett (1967). It appears to the author that these comments refer to four points: (1) energy transfer by muon-type neutrinos, (2) nucleon URCA process, (3) mass zoning used for numerical solution, and (4) core-shock structure. We have already discussed the first and will proceed to the remaining three points.

a) Nucleon URCA Process

The nucleon URCA process is of paramount importance in the development of the collapse of the core of a massive star. Preliminary investigation of the photodisintegration of ^{56}Fe by Truran and Arnett (1967) indicates that for higher densities ($\rho \gtrsim 10^9$ g cm^{-3}) an appreciable number of free protons are produced as the ^{56}Fe is destroyed. For densities ($\rho \sim 10^9$–10^{10} g cm^{-3}), the mean free life for a proton against electron capture becomes less than the free-fall time of the center of the star. This implies that the nuclear evolution for this stage cannot be properly treated without considering the coupling of the processes of photodisintegration and electron capture. CW ignored this effect and estimated the nucleon URCA rate by considering the process

$$p + e^- \rightarrow n + \nu_e$$

for a *fixed* composition. This could be drastically in error (the reduction of this process by neutron degeneracy causes improvement in the hydrodynamic behavior of a CW model, although the zoning is still not quite adequate). Unfortunately, a more sophisticated treatment of the nucleon URCA rates would probably be useless without a corresponding improvement in the treatment of neutrino energy transfer; the nucleon URCA rates are important when the mean free path for thermal neutrinos is of the order of or less than the dimensions of the region considered.

Colgate has pointed out that the nucleon URCA rate used by Arnett (1967) is an underestimate. This is the case; it was assumed that *if* an explosion were to occur, the deposition of energy by neutrinos from the core shock would overwhelm losses by the nucleon URCA. With this assumption it was found that densities at which nucleon URCA was very effective lay *inside* the neutrino-emission surface. This situation is self-consistent; the question is whether it may be reached by the *evolution* of a collapsing star. In order to determine this, the author has recalculated the evolution of an 8 $M_\odot$ stellar core, using Colgate's nucleon URCA rate (reduced for neutron degeneracy) and a corrected equation of state, and requiring that those zones in which the core shock occurred be treated in the diffusion approximation. The evolution begins like that of CW, but when neutrino energy transfer becomes important, it relaxes to that described in Arnett (1967). No explosion occurs. An attempt to repeat this calculation with a 2 $M_\odot$ stellar core merely convinced the author that a better mathematical treatment of energy transfer at low "optical" depths for neutrinos was needed.

b) Mass Zoning

In order to attack the problem of a collapsing star, the hydrodynamic equations were converted to a finite difference form and solved numerically. The system was Lagrangian so that the star was divided into concentric spherical shells of equal mass. Colgate has asserted that the number of such shells used by Arnett (1967) was inadequate to reproduce the density variations correctly. This was not the case, as is strikingly shown by

Figure 1, where each dot represents a separate zone in a late state of the calculation, when the zoning was at its *worst*. This calculation, using eighty mass zones, was indistinguishable from that using forty zones reported in Arnett (1967). Temperature and other variables vary as smoothly as the run of density shown in Figure 1. The model shown did not explode.

c) Shock Structure

In this section, the subtle question of the relationship between hydrodynamics and energy transfer will be investigated by analytic means *insofar as this is possible*. Instances of recourse to numerical results will be clearly indicated.

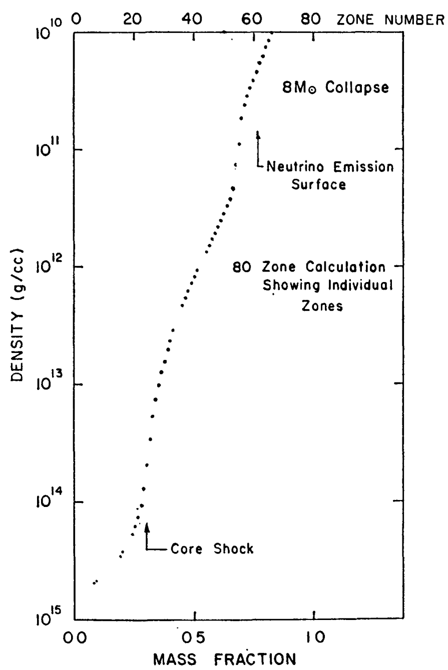

FIG. 1.—Density versus mass fraction at a late stage in the collapse of an 8 $M_\odot$ core. Each dot represents a separate zone. Such zoning allows accurate numerical calculations of hydrodynamic behavior. Physical assumptions are described in Arnett (1967).

The mutual potential energy of a spherical shell of mass dM_r and all shells interior to it is (Eddington 1926)

$$-d\Omega = \frac{GM_r}{r}\,dM_r\,. \tag{1}$$

If we assume that the density is roughly uniform interior to radius r, then

$$r \cong [3M_r/(4\pi\rho_{\text{core}})]^{1/3}\,. \tag{2}$$

In a free-fall collapse, the potential energy release is equal to the kinetic energy dJ of a mass element dM_r, so

$$\begin{aligned} dJ &= \tfrac{1}{2}U^2 dM \\ &= -d\Omega\,. \end{aligned} \tag{3}$$

Using equations (1), (2), and (3), we relate the velocity of infalling matter, U, to the mass and density of the dense quasi-static core onto which it falls:

$$U = [2GM_r^{2/3}(4\pi\rho_{\text{core}}/3)^{1/3}]^{1/2} . \tag{4}$$

As the radius r is the boundary between a quasi-static dense core and matter moving inward in free fall, a shock front develops at r. The flux of kinetic energy into this front is

$$F_{\text{KE}} = \rho_{ps}U^3/2 , \tag{5}$$

where ρ_{ps} is the density of matter before it enters the shock front. If we make the extreme assumption that all this energy is radiated in the form of electron-type neutrinos and antineutrinos, and if in addition we assume that the distribution function for these particles may be approximated by that of a fermion black body, then the neutrino and antineutrino flux is

$$F_\nu = \tfrac{7}{8}\sigma T^4 = F_{\text{KE}} , \tag{6}$$

where σ is Stefan's constant and helicity restrictions have been included. The temperature behind the shock is given by

$$T = [4M_r\rho_{ps}(2G)^{3/2}(4\pi\rho_{\text{core}}/3)^{1/2}/7\sigma]^{1/4} . \tag{7}$$

If we introduce the variables $m = M_r/M_\odot$, $\rho^* = \rho_{\text{core}}/(3 \times 10^{14}\ \text{g cm}^{-3})$, and $\eta = \rho_{\text{core}}/\rho_{ps}$, we have

$$T = (1.82 \times 10^{12}\ {}^\circ\,\text{K})\left(\frac{m}{\eta}\right)^{1/4} (\rho^*)^{3/8} . \tag{8}$$

If the initial contraction proceeds at low temperature, it will halt when nucleon-nucleon repulsion becomes important. This occurs at approximately nuclear density, so $\rho^* = 1$. In order to proceed further, we need information that is readily available only by use of numerical hydrodynamics. This was obtained from a calculation of the collapse of an 8 $M_\odot$ star, using Colgate's nucleon URCA rate (reduced for neutron degeneracy) and a corrected equation of state. During the early stages of the collapse (before the effects of pseudo-viscous pressure and of energy transfer are important), it is found that $\rho_{ps} \gtrsim 5 \times 10^{12}$ g cm^{-3} and $M_r \sim 0.1\ M_\odot$. The neutrino radiation temperature is then

$$T > 360 \times 10^9\ {}^\circ\,\text{K} . \tag{9}$$

This implies an average nucleon energy $1.5\,kT \gtrsim 45$ MeV, which contradicts our neglect of thermal pressure in estimating ρ_{core}. This tends to decrease ρ_{core}, but as more matter falls in, m increases. After the hot core grows to 0.4 $M_\odot$, the density decrease ($\rho_{\text{core}} \sim 4 \times 10^{13}$ g cm^{-3}) results in a neutrino radiation temperature T of 400×10^9 ° K. This agrees, more or less, with the previous estimate (9), which we adopt in what follows.

At these high temperatures, the processes which produce electron-type neutrino-antineutrino pairs occur at a prodigious rate. The neutron-rich gas now becomes less degenerate, so that the equilibrium composition favors more protons and electrons. Neutron decay is too slow ($\tau \sim 10^3$ sec) to be effective, but the process

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e \tag{10}$$

is not. Using the results of Bahcall and Wolf (1965) for the degenerate case, we find an upper limit on the time scale for reaction (10) to go to completion ($\tau \lesssim 5 \times 10^{-7}$ sec), which is much less than the collapse time scale ($\tau \sim 3$ to 5×10^{-3} sec).

An approximate form for the cross-section for electron-antineutrino scattering in a degenerate electron gas has been given by Bahcall (1964)

$$\sigma \cong \frac{\sigma_0}{3}\,\omega\epsilon_f\,, \qquad \omega \gg \epsilon_f \gg 1\,, \tag{11}$$

where $\sigma_0 = 2 \times 10^{-44}$ cm^2 and ω and ϵ_f are neutrino energy and electron Fermi energy in units of electron rest mass energy. Hansen (1966) has numerically evaluated the cross-section for this process in the case where the electron gas is semidegenerate also and has found that the approximate form is reasonably good in the range of interest here. At densities greater than $\rho \sim 10^{11}$ g cm^{-3}, electron capture will occur on all terrestrially stable nuclei, even at zero temperature. We obtain a lower limit for the electron number density if we take

$$n_{e-} = 6 \times 10^{34}\ \mathrm{cm}^{-3}\,, \tag{12}$$

which corresponds to an electron Fermi energy of 23 MeV. Taking the neutrino energy as $3kT$, we can evaluate the mean free path for antineutrino-electron scattering.

$$\begin{aligned}\lambda &= 3/(n_{e-}\sigma_0\omega\epsilon_f)\\ &= 3.0 \times 10^5\ \mathrm{cm}\,.\end{aligned} \tag{13}$$

The mean free path for neutrino-electron scattering is one-third of this value.

At this point we must again have recourse to the results of numerical hydrodynamics. The radius of that region for which the density was greater than 10^{11} g cm^{-3} (and in which most of the weak interaction opacity occurs) was found to be

$$R \gtrsim 1.3 \times 10^7\ \mathrm{cm}\,. \tag{14}$$

The value given was obtained at the moment of formation of a central region of nuclear density. To minimize the effect of the pseudo-viscous pressure, all zones with such pressure were taken to have zero width in determining (14). As the collapse continues, R increases. The radius R must be *much* larger than the radius of a neutron star of low temperature and the same mass M_r. (The work of Tsuruta and Cameron [1966] gives $r_{ns} \sim 2 \times 10^6$ cm for $M_r \sim 0.2\,M_\odot$). Colgate (1967) gives a value of R somewhat smaller than in equation (14); this may be due to the equation-of-state difficulty mentioned above. The radius R is much larger than the mean free path for antineutrino-electron scattering, $R/\lambda \sim 43$, and neutrino-electron scattering, $R/\lambda \sim 130$. The fraction of these particles which escapes without interaction (of order exp $[-R/\lambda]$) is very small.

The scattering is not elastic, however. We define the fractional energy change by antineutrino (neutrino) per scatter as

$$\left\langle\frac{\omega - \omega'}{\omega}\right\rangle = 1 - \xi\,.$$

For our conditions, $\xi \sim \frac{1}{2}$ to $\frac{3}{4}$.

From (13) we see that after the nth scatter, the mean free path is related to the antineutrino energy by

$$\lambda_n = \lambda_{n-1}\omega_n/\omega_{n-1}\,.$$

The distance traveled after n encounters is

$$l_n = \sum_{i=1}^{n}\lambda_i\,.$$

Escape from the region interior to R occurs, on the average, when $\lambda_n \sim R$. Numerical evaluation indicates that this occurs when $l_n \sim R$ to a factor of 2 or so. Assuming that the fractional energy change per scatter is constant, we find that the time for antineutrino escape to low density regions is

$$\tau \simeq \begin{cases} 1.3 \times 10^{-3} \text{ sec}, & \xi = \frac{1}{2} \\ 2.2 \times 10^{-3} \text{ sec}, & \xi = \frac{3}{4}, \end{cases} \tag{15}$$

and that the antineutrino energy has been degraded to about 1.5 MeV in both cases. This is slightly less than the thermal energy of matter at a density $\rho \lesssim 10^{11}$ g cm^{-3}, which is undergoing ^{4}He photodisintegration. Thus we see that the antineutrino flux has been thermalized and most of its energy deposited in the density range

$$10^{11} < \rho < 10^{14} \text{ g cm}^{-3} .$$

The time scale for antineutrino escape (eq. [15]) is only slightly less than the free-fall time scale. The matter in this density range cannot re-radiate the antineutrino-deposited energy while it is degenerate, so it heats up. *This condition also implies that thermal pressure effects cannot be neglected.*

As the matter heats up, its net opacity to antineutrinos (neutrinos) increases from three effects: (1) the cross-section is proportional to electron energy, which increases; (2) more electrons and protons are formed, so that equation (12) drastically underestimates the electron number density; and (3) the thermal pressure acts in such a way as to increase the radius R above the value given by equation (14). The time scale for antineutrino (neutrino) escape becomes greater than that for collapse, and much of the energy release from gravitational contraction goes into low-energy ($\epsilon < m_\mu c^2$) muon-type neutrinos and antineutrinos which readily escape. This is the sort of behavior found by Arnett (1967). Thus we see that the core-shock structure proposed by CW is not stable for sufficiently massive stars (i.e., those with a sufficiently high flux of kinetic energy incident on the dense core) and tends to a somewhat different structure. Consequently, the mass-zoning requirements estimated by Colgate (1967) are unnecessarily restrictive. Similar analysis applied to the situation described above indicates that the zoning used by Arnett (1967) was adequate for an exploratory calculation.

III. CONCLUSION

A proper treatment of the gravitational collapse of non-rotating massive stars involves the coupling of (1) hydrodynamics, (2) energy transport by neutrinos, and (3) compositional change. Item (3) is related to (2) by opacity and to (1) by the equation of state, which in turn reacts back on (3) through temperature and density. The primary difference in the nature of the results of CW and Arnett (1967) is due to the fact that, in the former calculation, items (1) and (2) were not properly coupled in the density range

$$10^{11} < \rho < 10^{14} \text{ g cm}^{-1} .$$

Their particular choice of equation of state and energy transport procedure conspired to obscure this neglect, which became apparent in an alternative method of attacking the problem.

As yet the even more delicate problem of including the coupling of item (3) with (1) and (2) has not been attacked in a satisfactory manner.

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REFERENCES

Arnett, W. D. 1967, *Canadian J. Phys.*, **45**, 1621.
Bahcall, J. N. 1964, *Phys. Rev.*, **136**, B1164.
Bahcall, J. N., and Wolf, R. A. 1965, *Phys. Rev.*, **140**, B1452.
Colgate, S. A. 1968, *Ap. J.*, **153**, 335.
Colgate, S. A., and White, R. 1966, *Ap. J.*, **143**, 626.
Eddington, A. S. 1926, *The Internal Constitution of the Stars* (Cambridge: Cambridge University Press), p. 86.
Hansen, C. J. 1966 (preprint).
———. 1967 (private communication).
Salpeter, E. E. 1960, *Ann. Phys.*, **11**, 393.
Truran, J. W., and Arnett, W. D. 1967 (in preparation).
Tsuruta, S., and Cameron, A. G. W. 1966, *Canadian J. Phys.*, **44**, 1895.

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